CH-2 Polynomial

Question: 1. Use suitable identities to find the following products:

(i) (x + 4) (x + 10)

Solution:- *(x+4)(x+10)*

$= x^2+10x+4x+4 \times 10$

$= x^2+14x+40$

(ii) (x + 8) (x – 10)

Solution: $x^2-10x+8x-80$

$= x^2-2x-80$

(iii) (3x + 4) (3x – 5)

Solution: $9x^2-15x+12x-20$

$= 9x^2-3x-20$

(iv) $(y^2 + \frac{3}{2})(y^2 - \frac{3}{2})$

Solution: $(a+b)(a-b) = a^2-b^2$

Suppose, $y^2=a$ and $\frac{3}{2}=b$

Hence, the given equation can be written as follows:

$$y^4 - \frac{9}{4}$$

(v) (3 – 2x) (3 + 2x)

Solution: This can be solved as the earlier question

$(3-2x)(3+2x) = 9-4x^2$

2. Evaluate the following products without multiplying directly:

(i) 103 × 107

Solution: 103 × 107

= (100+3)(100+7)

= 1002+7 × 100+3 × 100+7 × 3

= 10000+700+300+21

= 11021

(ii) 95 × 96

Solution: 95 × 96

= (100-5)(100-4)

= 1002-400-500+20

= 10000-900+20

= 9120

(iii) 104 × 96

Solution: 104 × 96

= (100+4)(100-4)

= 1002-42

= 10000-16

= 9984

3. Factorise the following using appropriate identities:

(i) $9x^2 + 6xy + y^2$

Solution: $9x^2+6xy+y^2$

$= 3^2x^2+3xy+3xy+y^2$ …………………………… (1)

$= 3x(3x+y)+y(3x+y)$

$= (3x+y)(3x+y)$

$= (3x+y)^2$

Alternate way of solving this problem:

Equation 1 gives a hint that this can be solved through following formula:

$(a+b)^2 = a^2+2ab+b^2$

(ii) $4y^2 – 4y + 1$

Solution: $2^2y^2-2 \times 2y+1^2$

$= (2y-1)^2$

(iii) $x^2 - \frac{y^2}{100}$

Solution: As you know $a^2-b^2 = (a+b)(a-b)$

So, $x^2 - \frac{y^2}{10^2}$

$= (x+\frac{y}{10})(x-\frac{y}{10})$

4. Expand each of the following, using suitable identities:

(i) $(x + 2y + 4z)^2$

Solution: As you know $(x + y + z)^2= x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Using this formula in the given equation,

$(x+2y+4z)^2$

$= x^2+4y^2+16z^2+4xy+16yz+8zx$

(ii) ***(2x – y + z)***^{2}

Solution: $(x-y+z)^2 = x^2+y^2+z^2-2xy-2yz+2zx$

So, $(2x-y+z)^2$

$= 4x^2+y^2+z^2-4xy-2yz+4zx$

(iii) $(-2x + 3y + 2z)^2$

Solution: $(-2x+3y+2z)^2$

$= 4x^2+9y^2+4z^2-12xy+12yz-8zx$

(iv) $(3a - 7b - c)^2$

Solution: $(x-y-z)^2= x^2+y^2+z^2-2xy-2yz-2zx$

Hence, $(3a-7b-c)^2$

$= 9a^2+49b^2+c^2-42ab-14bc-6ac$

5. Factorise:

(i) **$4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$**

Solution: It is clear that this can be solved using

$(x+y-z)^2 = x^2+y^2+z^2+2xy-2yz-2zx$

Hence, $4x^2+9y^2+16z^2+12xy-24yz-2zx$

$= (2x+3y-4z)^2$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

Solution: using $(x+y+z)^2$

The value of $x= \sqrt{2}x$, $y=y$ and $z= 2\sqrt{2}z$

Hence, given equation can be written as follows:

$(\sqrt{2}x+y+2\sqrt{2}z)^2$

6. Write the following cubes in expanded form:

(i) $(2x + 1)^3$

Solution: As you know, $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

Hence, $(2x+1)^3$

$= 8x^3+1+6xy(2x+1)$

(ii) $(2a – 3b)^3$

Solution: As you know, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$

Hence, $(2a-3b)^3 = 8a^3-27y^3-18ab(2a-3b)$

(iii) $(\frac{3}{2}x+1)^3$

Solution: Using the formula which was used in question 6-(i)

We get, $\frac{27}{8}x^3+1+\frac{9}{2}xy(\frac{3}{2}x+y)$

7. Evaluate the following:

(i) 99^3

Solution: 99^3 can be written as $(100-1)^3$

$(100-1)^3$ can be solved through using $(x-y)^3$

Now, $(100-1)^3= 100^3-1^3-300(100-1)$

$= 1000000-1-300(99)$

$= 1000000-1-29700$

$= 970299$

(ii) 102^3

Solution: 102^3 can be written as $(100+2)^3$ and can be solved using $(x+y)^3$

Hence, $(100+2)^3$

$= 100^3+2^3+600(100+2)$

$= 1000000+8+61200$

$= 1061208$

8. Factorise each of the following:

***(i)* $8a^3 + b^3 + 12a^2b + 6ab^2$**

Solution:

$= 8a^3+b^3+6ab(a+b)$

$=(2a+b)^3$

***(ii)* $8a^3 – b^3 – 12a^2b + 6ab^2$**

Solution:

$8a^3-b^3-12a^2b+6ab^2$

$=8a^3-b^3-6ab(a+b)$

$=(2a-b)^3$

***(iii)* $27 – 125a^3 – 135a + 225a^2$**

Solution: $27 – 125a^3 – 135a + 225a^2$

$= 3^3-5^3a^3-3^35a+3^25^2a^2$

$= 3^3-5^3a^3-3^25a(3-5a)$

if x=3 and y=5a

hence, $(3-5a)^3= 3^3-5^3a^3-3^25a(3-5a)$

(iv) $64a^3 – 27b^3 – 144a^2b + 108ab^2$

Solution: *$64a^3 – 27b^3 – 144a^2b + 108ab^2$*

= $4^3a^3-3^3b^3-3 \times 4a3b(4a-3b)$

= $(4a-3b)^3$

Note: Try to identify the values of x and y by carefully analysing the first two terms of the equations. This will give you exact clue to the final answer.

(v) $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$

Solution: $27p^3$ can be written as 3^3p^3

Hence, x= 3p

And $\frac{1}{216}$ can be written as $\frac{1^3}{6^3}$

Hence, y= $\frac{1}{6}$

So, the required answer will be $(3p-\frac{1}{6})^3$

Note: This step is to help you develop the problem solving skills. In exam situation you have to write all steps to get full marks.

9. Verify :

(i) $x^3 + y^3 = (x + y) (x^2 – xy + y^2)$

Solution: RHS $(x+y)(x^2-xy+y^2)$

= $x^3-x^2y+xy^2+x^2y-xy^2+y^3$

=x^3+y^3 LHS proved

(ii) $x^3 – y^3 = (x – y) (x^2 + xy + y^2)$

Solution: RHS $(x – y) (x^2 + xy + y^2)$

= $x^3+x^2y+xy^2-x^2y-xy^2-y^3$

= x^3-y^3 LHS proved

10. Factorise each of the following:

(i) $27y^3 + 125z^3$

Solution: From the previous question you can recall

$x^3+y^3=(x+y)(x^2+xy+y^2)$

Hence, $3^3y^3+5^3z^3$ can be written as follows: ($27=3^3$ and $125=5^3$)

$(3y+5z)(9y^2+15yz+25z^2$

(ii) $64m^3 – 343n^3$

Solution: As you know, $x^3-y^3=(x-y)(x^2+xy+y^2)$

Hence, $4^3m^3-7^3n^3$ can be written as follows: ($64=4^3$ and $343=7^3$)

$4m-7n)(16n^2+28mn+49n^2)$

11. Factorise : *$27x^3 + y^3 + z^3 – 9xyz$*

Solution: As you know,

$x^3 + y^3 + z^3 – 3xyz = (x + y + z) (x^2 + y^2 + z^2 – xy – yz – zx)$

Hence, $27x^3 + y^3 + z^3 – 9xyz$

$= (3x+y+z) (9x^2+y^2+z^2-3xy-yz-3zx)$

12. Verify that

$$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x+y+z) [(x-y)^2+(y-z)^2+(z-x)^2]$$

Solution: RHS $\frac{1}{2}(x+y+z) [(x-y)^2+(y-z)^2+(z-x)^2]$

$= \frac{1}{2}(x+y+z)(x^2+y^2-2xy+y^2+z^2-2yz+z^2+x^2-2zx)$

$= \frac{1}{2}(x+y+z) (2x^2+2y^2+2z^2-2xy-2yz-2zx)$

$=(x+y+z) (x^2+y^2+z^2-xy-yz-zx)$

Now, the RHS satisfies the condition as per following identity:
$x^3 + y^3 + z^3 - 3xyz = (x + y + z) (x^2 + y^2 + z^2 - xy - yz - zx)$

13. If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$.

Solution: As you know,

$x^3 + y^3 + z^3 – 3xyz = (x + y + z) (x^2 + y^2 + z^2 – xy – yz – zx)$

Now, as per question x+y+z=0,

Putting value of x+y+z=0 in the equation we get

$x^3 + y^3 + z^3 – 3xyz = 0 (x^2 + y^2 + z^2 – xy – yz – zx)$

Or, $x^3 + y^3 + z^3 – 3xyz = 0$

Or, $x^3 + y^3 + z^3 = 3xyz$ proved

14. Without actually calculating the cubes, find the value of each of the following:

(i) $(–12)^3 + (7)^3 + (5)^3$

Solution: As you know,

$x^3 + y^3 + z^3 – 3xyz = (x + y + z) (x^2 + y^2 + z^2 – xy – yz – zx)$

Or, $x^3 + y^3 + z^3 =(x + y + z) (x^2 + y^2 + z^2 – xy – yz – zx)+3xyz$

Hence, $(–12)^3 + (7)^3 + (5)^3$

$=(-12+7+5) [-12^2+7^2+5^2-(-12 \times 7)-(7 \times 5)-(-12 \times 5)]+3(-12 \times 7 \times 5)$

$=0 -12^2+7^2+5^2[-(-12 \times 7)-(7 \times 5)-(-12 \times 5)] -1260$

$= 0-1260 = -1260$ answer

(ii) $(28)^3 + (–15)^3 + (–13)^3$

Solution: This question can be solved in the same way as above.

Here, value of (x+y+z) = (28-15-13) = 0

Hence, you need to calculate the value of 3xyz

3 x 28 x -15 x -13 = 16380

Hence, the required answer = -16380