CH – 12   HERON'S FORMULA

Q.1: A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board?

Solution: Side of traffic signal board = a

Perimeter of traffic signal board = $3 \times a$

$$2s = 3a \Rightarrow s = \frac{3}{2}a$$

By Heron's formula,

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{Area of given triangle} = \sqrt{\frac{3}{2}a\left(\frac{3}{2}a-a\right)\left(\frac{3}{2}a-a\right)\left(\frac{3}{2}a-a\right)}$$

$$= \sqrt{\left(\frac{3}{2}a\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)}$$

$$= \frac{\sqrt{3}}{4}a^2$$

Perimeter of traffic signal board = 180 cm

$$(a) = \left(\frac{180}{3}\right)cm = 60\,cm$$

Side of traffic signal board

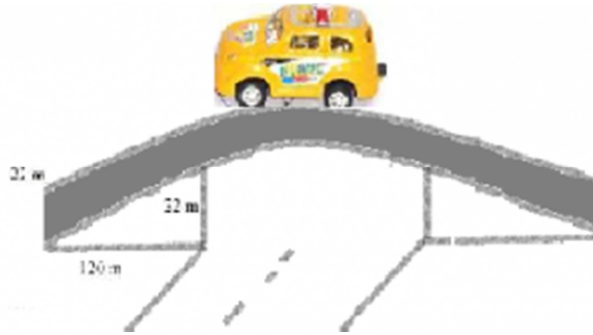Using equation (1), area of traffic signal board $= \frac{\sqrt{3}}{4}(60\ cm)^2$

$$= \left(\frac{3600}{4}\sqrt{3}\right)cm^2 = 900\sqrt{3}\ cm^2$$

Q.2: The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122m, 22m, and 120m (see the given figure). The advertisements yield an earning of Rs 5000 per m^2 per year. A company hired one of its walls for 3 months. How much rent did it pay?

Solution: The sides of the triangle (i.e., *a*, *b*, *c*) are of 122 m, 22 m, and 120 m respectively.

Perimeter of triangle = (122 + 22 + 120) m

$2s$ = 264 m

s = 132 m

By Heron's formula,

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{Area of given triangle} = \left[\sqrt{132(132-122)(132-22)(132-120)}\right] m^2$$

$$= \left[\sqrt{132(10)(110)(12)}\right] m^2 = 1320 \ m^2$$

Rent of 1 m² area per year = Rs 5000

Rent of 1 m² area per month = Rs $\dfrac{5000}{12}$

Rent of 1320 m² area for 3 months = Rs $\left(\dfrac{5000}{12} \times 3 \times 1320\right)$

= Rs (5000 × 330) = Rs 1650000

Therefore, the company had to pay Rs 1650000.

Q.3: There is a slide in the park. One of its side walls has been painted in the same colour with a message "KEEP THE PARK GREEN AND CLEAN" (see the given figure). If the sides of the wall are 15m, 11m, and 6m, find the area painted in colour.

Solution: It can be observed that the area to be painted in colour is a triangle, having its sides as 11 m, 6 m, and 15 m.

Perimeter of such a triangle = $(11 + 6 + 15)$ m

$2\,s = 32$ m

$s = 16$ m

By Heron's formula,

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \left[\sqrt{16(16-11)(16-6)(16-15)}\right] m^2$$
$$= \left(\sqrt{16 \times 5 \times 10 \times 1}\right) m^2$$
$$= 20\sqrt{2}\ m^2$$

Therefore, the area painted in colour is . $20\sqrt{2}\ m^2$

Q.4: Find the area of a triangle two sides of which are 18 cm and 10 cm and the perimeter is 42 cm.

Solution: Let the third side of the triangle be x.

Perimeter of the given triangle = 42 cm

18 cm + 10 cm + x = 42

x = 14 cm

$$s = \frac{\text{Perimeter}}{2} = \frac{42\ cm}{2} = 21\ cm$$

By Heron's formula,

$$\text{Area of a triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$\text{Area of the given triangle} = \left(\sqrt{21(21-18)(21-10)(21-14)}\right) cm^2$$
$$= \left(\sqrt{21(3)(11)(7)}\right) cm^2$$
$$= 21\sqrt{11}\ cm^2$$

Q.5: Sides of a triangle are in the ratio of 12: 17: 25 and its perimeter is 540 cm. Find its area ?

Solution: Let the common ratio between the sides of the given triangle be x.

Therefore, the side of the triangle will be $12x$, $17x$, and $25x$.

Perimeter of this triangle = 540 cm

$12x + 17x + 25x = 540$ cm

$54x = 540$ cm

$x = 10$ cm

Sides of the triangle will be 120 cm, 170 cm, and 250 cm.

$$s = \frac{\text{Perimeter of triangle}}{2} = \frac{540 \text{ cm}}{2} = 270 \text{ cm}$$

By Heron's formula,

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \left[\sqrt{270(270-120)(270-170)(270-250)} \right] \text{cm}^2$$
$$= \left[\sqrt{270 \times 150 \times 100 \times 20} \right] \text{cm}^2$$
$$= 9000 \text{ cm}^2$$

Therefore, the area of this triangle is 9000 cm².

Q.6: An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.

Solution: Let the third side of this triangle be x.

Perimeter of triangle = 30 cm

12 cm + 12 cm + x = 30 cm

$x = 6$ cm

$$s = \frac{\text{Perimeter of triangle}}{2} = \frac{30 \text{ cm}}{2} = 15 \text{ cm}$$

By Heron's formula,

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \left[\sqrt{15(15-12)(15-12)(15-6)} \right] \text{cm}^2$$
$$= \left[\sqrt{15(3)(3)(9)} \right] \text{cm}^2$$
$$= 9\sqrt{15} \text{ cm}^2$$